The purpose of this notebook is to verify that a certain pair of circuits have (almost) the same effect on the input state |000..0>.

These two lines immediately below only need to be executed once, after Mathematica launch

Download the QuEST Mathematica package, and
Connect to the remote Igor server or Local

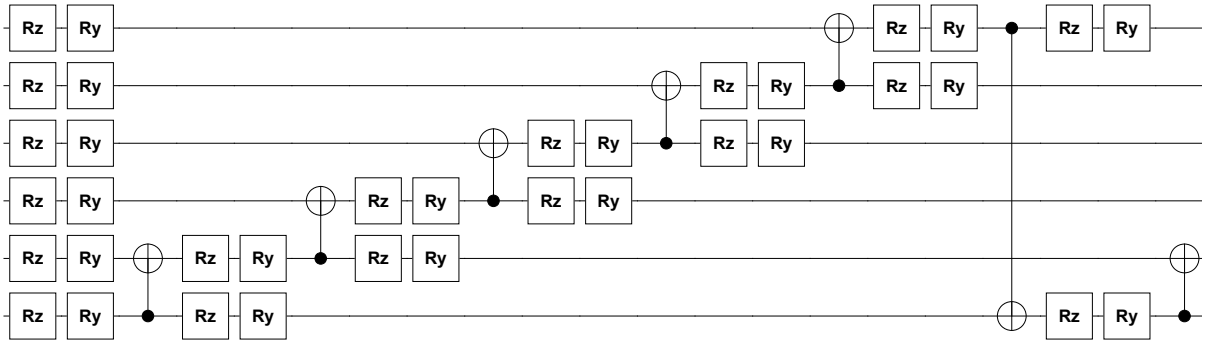
In[132]:=
```
Import["https://quest.qtechtheory.org/QuEST.m"];
CreateDownloadedQuESTEnv[];
```

Build the circuit found in the main part of the paper (258 gates, 204 with general parameters)

In[134]:=
```
circuitA = {Rz0[α1], Rz1[α3], Rz2[α5], Rz3[α7], Rz4[α9], Rz5[α11], Ry0[α2], Ry1[α4],
    Ry2[α6], Ry3[α8], Ry4[α10], Ry5[α12], C0[X1], Rz0[α13], Rz1[α15], Ry0[α14],
    Ry1[α16], C1[X2], Rz1[α17], Rz2[α19], Ry1[α18], Ry2[α20], C2[X3], Rz2[α21], Rz3[α23],
    Ry2[α22], Ry3[α24], C3[X4], Rz3[α25], Rz4[α27], Ry3[α26], Ry4[α28], C4[X5],
    Rz4[α29], Rz5[α31], Ry4[α30], Ry5[α32], C5[X0], Rz5[α33], Rz0[α35], Ry5[α34],
    Ry0[α36], C0[X1], Rz0[α37], Rz1[α39], Ry0[α38], Ry1[α40], C1[X2], Rz1[α41],
    Rz2[α43], Ry1[α42], Ry2[α44], C2[X3], Rz2[α45], Rz3[α47], Ry2[α46], Ry3[α48],
    C3[X4], Rz3[α49], Rz4[α51], Ry3[α50], Ry4[α52], C4[X5], Rz4[α53], Rz5[α55],
    Ry4[α54], Ry5[α56], C5[X0], Rz5[α57], Rz0[α59], Ry5[α58], Ry0[α60], C0[X1],
    Rz0[α61], Rz1[α63], Ry0[α62], Ry1[α64], C1[X2], Rz1[α65], Rz2[α67], Ry1[α66],
    Ry2[α68], C2[X3], Rz2[α69], Rz3[α71], Ry2[α70], Ry3[α72], C3[X4], Rz3[α73],
    Rz4[α75], Ry3[α74], Ry4[α76], C4[X5], Rz4[α77], Rz5[α79], Ry4[α78], Ry5[α80],
    C5[X0], Rz5[α81], Rz0[α83], Ry5[α82], Ry0[α84], C0[X1], Rz0[α85], Rz1[α87],
    Ry0[α86], Ry1[α88], C1[X2], Rz1[α89], Rz2[α91], Ry1[α90], Ry2[α92], C2[X3],
    Rz2[α93], Rz3[α95], Ry2[α94], Ry3[α96], C3[X4], Rz3[α97], Rz4[α99], Ry3[α98],
    Ry4[α100], C4[X5], Rz4[α101], Rz5[α103], Ry4[α102], Ry5[α104], C5[X0], Rz5[α105],
    Rz0[α107], Ry5[α106], Ry0[α108], C0[X1], Rz0[α109], Rz1[α111], Ry0[α110], Ry1[α112],
    C1[X2], Rz1[α113], Rz2[α115], Ry1[α114], Ry2[α116], C2[X3], Rz2[α117], Rz3[α119],
    Ry2[α118], Ry3[α120], C3[X4], Rz3[α121], Rz4[α123], Ry3[α122], Ry4[α124], C4[X5],
    Rz4[α125], Rz5[α127], Ry4[α126], Ry5[α128], C5[X0], Rz5[α129], Rz0[α131], Ry5[α130],
    Ry0[α132], C0[X1], Rz0[α133], Rz1[α135], Ry0[α134], Ry1[α136], C1[X2], Rz1[α137],
    Rz2[α139], Ry1[α138], Ry2[α140], C2[X3], Rz2[α141], Rz3[α143], Ry2[α142], Ry3[α144],
    C3[X4], Rz3[α145], Rz4[α147], Ry3[α146], Ry4[α148], C4[X5], Rz4[α149], Rz5[α151],
    Ry4[α150], Ry5[α152], C5[X0], Rz5[α153], Rz0[α155], Ry5[α154], Ry0[α156], C0[X1],
    Rz0[α157], Rz1[α159], Ry0[α158], Ry1[α160], C1[X2], Rz1[α161], Rz2[α163], Ry1[α162],
    Ry2[α164], C2[X3], Rz2[α165], Rz3[α167], Ry2[α166], Ry3[α168], C3[X4], Rz3[α169],
    Rz4[α171], Ry3[α170], Ry4[α172], C4[X5], Rz4[α173], Rz5[α175], Ry4[α174], Ry5[α176],
    C5[X0], Rz5[α177], Rz0[α179], Ry5[α178], Ry0[α180], C0[X1], Rz0[α181], Rz1[α183],
    Ry0[α182], Ry1[α184], C1[X2], Rz1[α185], Rz2[α187], Ry1[α186], Ry2[α188], C2[X3],
    Rz2[α189], Rz3[α191], Ry2[α190], Ry3[α192], C3[X4], Rz3[α193], Rz4[α195], Ry3[α194],
    Ry4[α196], C4[X5], Rz4[α197], Rz5[α199], Ry4[α198], Ry5[α200], C5[X0], Rz5[α201],
    Rz0[α203], Ry5[α202], Ry0[α204], C5[X0], C4[X5], C3[X4], C2[X3], C1[X2], C0[X1]};

DrawCircuit[circuitA /. αq_ → α]
(* note: the replacement rule αq_→
 α strips all the indices from the parameters;
otherwise DrawCircuit will think they refer to qubits
 and conclude there are 205 qubits in the circuit! *)
```

Out[135]=



In[136]:=

```
paramsA = {α1 → 0.0101334`, α2 → 0.769441`, α3 → 0.011897`, α4 → 0.583183`,
    α5 → 0.00456617`, α6 → -0.145715`, α7 → 0.00777718`, α8 → 0.119862`,
    α9 → 0.0175044`, α10 → -0.355461`, α11 → 0.0179853`, α12 → 0.116027`,
    α13 → 0.216869`, α14 → 0.173293`, α15 → 0.162301`, α16 → 0.579826`,
    α17 → -0.0190162`, α18 → 0.342455`, α19 → 0.0244839`, α20 → 0.079771`,
    α21 → 0.0644038`, α22 → 0.250365`, α23 → -0.114621`, α24 → -0.223962`,
    α25 → -0.294749`, α26 → -1.04329`, α27 → 0.0844322`, α28 → 0.0371257`,
    α29 → 0.0163358`, α30 → -0.135934`, α31 → 0.172338`, α32 → 0.442457`,
    α33 → 0.0961484`, α34 → 0.566212`, α35 → -0.0271425`, α36 → -0.139685`,
    α37 → 0.0690098`, α38 → -0.18582`, α39 → -0.00437483`, α40 → 0.384239`,
    α41 → 0.0971316`, α42 → 0.121081`, α43 → 0.131357`, α44 → -0.242564`,
    α45 → -0.0554834`, α46 → -0.173258`, α47 → -0.290403`, α48 → 0.260711`,
    α49 → -0.339871`, α50 → 0.056221`, α51 → -0.00785937`, α52 → -0.394306`,
    α53 → 0.202649`, α54 → 0.0809169`, α55 → -0.0341023`, α56 → 0.0783848`,
    α57 → 0.0704627`, α58 → 0.660633`, α59 → -0.37343`, α60 → -0.335531`,
    α61 → -0.179003`, α62 → 0.272085`, α63 → 0.146456`, α64 → 0.273905`,
    α65 → 0.378093`, α66 → 0.244209`, α67 → 0.140811`, α68 → 0.00423213`,
    α69 → 0.277854`, α70 → 0.0800073`, α71 → -0.131185`, α72 → -0.0260716`,
    α73 → 0.0163183`, α74 → 0.020788`, α75 → 0.23544`, α76 → 0.541989`,
    α77 → -0.241904`, α78 → 0.299794`, α79 → -0.359987`, α80 → 0.182357`,
    α81 → -0.0662789`, α82 → -0.24588`, α83 → -0.277705`, α84 → -0.329012`,
    α85 → -0.20955`, α86 → -0.0703505`, α87 → -0.0623473`, α88 → 0.132676`,
    α89 → -0.0686734`, α90 → 0.602256`, α91 → 0.273906`, α92 → 0.360199`,
    α93 → 0.251797`, α94 → -1.00072`, α95 → 0.558969`, α96 → -0.278395`,
    α97 → 0.593785`, α98 → -0.670096`, α99 → -0.0602257`, α100 → -0.0659716`,
    α101 → -0.162023`, α102 → -0.350028`, α103 → 0.394134`, α104 → 0.113879`,
    α105 → 0.276657`, α106 → -0.392753`, α107 → 0.179687`, α108 → 0.00373736`,
    α109 → 0.129615`, α110 → 0.0719317`, α111 → -0.0503462`, α112 → -0.061521`,
    α113 → 0.0263394`, α114 → -0.187431`, α115 → -0.137458`, α116 → -0.303968`,
    α117 → -0.260625`, α118 → -0.0779235`, α119 → -0.100232`, α120 → 0.00133515`,
    α121 → -0.100561`, α122 → -0.00289775`, α123 → 0.0797794`, α124 → -0.459437`,
    α125 → 0.380958`, α126 → 0.633001`, α127 → 0.182504`, α128 → -0.398819`,
    α129 → 0.221514`, α130 → 0.0806666`, α131 → 0.149136`, α132 → 0.614864`,
    α133 → -0.550736`, α134 → -0.277595`, α135 → -0.0570828`, α136 → 0.124579`,
    α137 → 0.0327445`, α138 → -0.280098`, α139 → 0.0766052`, α140 → 0.571283`,
    α141 → 0.507224`, α142 → -0.370586`, α143 → -0.12298`, α144 → -0.346894`,
```

$\alpha_{145} \to -0.00400774`, \alpha_{146} \to -0.550065`, \alpha_{147} \to -0.414966`, \alpha_{148} \to 0.412483`,$
$\alpha_{149} \to -0.265857`, \alpha_{150} \to -0.145768`, \alpha_{151} \to 0.143879`, \alpha_{152} \to 0.232461`,$
$\alpha_{153} \to 0.153673`, \alpha_{154} \to 0.00700872`, \alpha_{155} \to 0.127383`, \alpha_{156} \to 0.265105`,$
$\alpha_{157} \to 0.0346656`, \alpha_{158} \to -0.0225024`, \alpha_{159} \to 0.532427`, \alpha_{160} \to -0.142008`,$
$\alpha_{161} \to 0.246349`, \alpha_{162} \to 0.0864951`, \alpha_{163} \to 0.206927`, \alpha_{164} \to -0.520599`,$
$\alpha_{165} \to 0.412435`, \alpha_{166} \to -0.152197`, \alpha_{167} \to 0.0240198`, \alpha_{168} \to -0.0727568`,$
$\alpha_{169} \to -0.121871`, \alpha_{170} \to 1.08538`, \alpha_{171} \to -0.105958`, \alpha_{172} \to 0.236211`,$
$\alpha_{173} \to 0.103953`, \alpha_{174} \to 0.306895`, \alpha_{175} \to -0.0336874`, \alpha_{176} \to 0.181208`,$
$\alpha_{177} \to -0.0987573`, \alpha_{178} \to 0.218696`, \alpha_{179} \to -0.138082`, \alpha_{180} \to 0.0751017`,$
$\alpha_{181} \to -0.0269803`, \alpha_{182} \to 0.450293`, \alpha_{183} \to -0.194875`, \alpha_{184} \to 0.177881`,$
$\alpha_{185} \to -0.166449`, \alpha_{186} \to 0.245931`, \alpha_{187} \to 0.16835`, \alpha_{188} \to 0.0886022`,$
$\alpha_{189} \to 0.207958`, \alpha_{190} \to -0.0703105`, \alpha_{191} \to 0.100214`, \alpha_{192} \to 0.0604428`,$
$\alpha_{193} \to 0.178772`, \alpha_{194} \to -0.343041`, \alpha_{195} \to 0.0825788`, \alpha_{196} \to 0.208969`,$
$\alpha_{197} \to -0.174065`, \alpha_{198} \to -0.388204`, \alpha_{199} \to -0.0438657`, \alpha_{200} \to -0.433672`,$
$\alpha_{201} \to -0.311434`, \alpha_{202} \to 0.335162`, \alpha_{203} \to -0.460915`, \alpha_{204} \to -0.160498`\};$

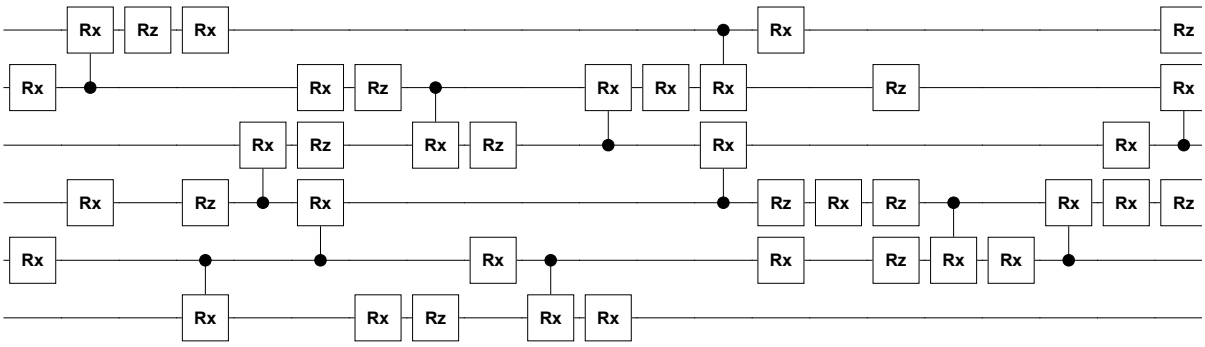
Now build an alternative circuit, found using a recompiler (124 gates, 124 with general parameters)

In[137]:=

```
circuitB = {Rx4[β1], Rx1[β2], C4[Rx5[β3]], Rx2[β4], Rz5[β5], Rx5[β6], Rz2[β7],
    C1[Rx0[β8]], C2[Rx3[β9]], Rz3[β10], C1[Rx2[β11]], Rx4[β12], Rz4[β13],
    Rx0[β14], C4[Rx3[β15]], Rz0[β16], Rz3[β17], Rx1[β18], C1[Rx0[β19]], Rx0[β20],
    C3[Rx4[β21]], Rx4[β22], C5[Rx4[β23]], C2[Rx3[β24]], Rx5[β25], Rz2[β26], Rx1[β27],
    Rx2[β28], Rz2[β29], Rz1[β30], Rz4[β31], C2[Rx1[β32]], Rx1[β33], C1[Rx2[β34]],
    Rx2[β35], Rx3[β36], C3[Rx4[β37]], Rz2[β38], Rz5[β39], Rx3[β40], Rz0[β41],
    C4[Rx5[β42]], C3[Rx4[β43]], Rx4[β44], Rx1[β45], C0[Rx1[β46]], C5[Rx4[β47]],
    Rz4[β48], C1[Rx0[β49]], Rz1[β50], Rx3[β51], C0[Rx1[β52]], C4[Rx3[β53]],
    Rx4[β54], Rx5[β55], Rz4[β56], Rx3[β57], Rz5[β58], C3[Rx2[β59]], Rx3[β60],
    Rz2[β61], C5[Rx4[β62]], Rz4[β63], Rx0[β64], Rz3[β65], Rz5[β66], C4[Rx5[β67]],
    C3[Rx2[β68]], C3[Rx4[β69]], Rz2[β70], C5[Rx4[β71]], C2[Rx1[β72]], Rx2[β73],
    Rz3[β74], Rx3[β75], Rz0[β76], C3[Rx4[β77]], C1[Rx0[β78]], Rx1[β79], Rx0[β80],
    Rx4[β81], Rz1[β82], C5[Rx4[β83]], C1[Rx2[β84]], C2[Rx1[β85]], C3[Rx2[β86]],
    Rz2[β87], Rz3[β88], Rx5[β89], Rx3[β90], Rx2[β91], C1[Rx2[β92]], Rx1[β93],
    Rz1[β94], C4[Rx5[β95]], Rx1[β96], C0[Rx1[β97]], Rz5[β98], Rz0[β99], Rx5[β100],
    Rz2[β101], C1[Rx0[β102]], Rz1[β103], Rx2[β104], Rx4[β105], C2[Rx1[β106]], Rx0[β107],
    Rz0[β108], C1[Rx2[β109]], Rx1[β110], Rz4[β111], Rx0[β112], Rx4[β113], Rz3[β114],
    Rz2[β115], C2[Rx1[β116]], C4[Rx5[β117]], C1[Rx2[β118]], C3[Rx4[β119]], Rx3[β120],
    C2[Rx1[β121]], Rx1[β122], Rz5[β123], Rz2[β124], C2[Rx3[β125]], C3[Rx4[β126]]};

DrawCircuit[circuitB /. βq_ → β]
```

Out[138]=



In[139]:=

```
paramsB = {β1 → 2.4217597714272787`, β2 → -2.056375118552786`,
    β3 → -7.499926599784914`, β4 → -1.596542698809316`, β5 → -0.6753639898521223`,
    β6 → 0.7948847392621772`, β7 → -4.0828115766793545`, β8 → -2.8539359720981863`,
    β9 → -2.6241090831292455`, β10 → -5.026653000544547`,
    β11 → -3.0139199991366596`, β12 → -8.23636085752444`, β13 → 3.8176491764469462`,
    β14 → -2.892717334427455`, β15 → 2.935802170001685`, β16 → 5.032339125790513`,
    β17 → -1.7569013695623599`, β18 → -5.144465980070199`,
    β19 → 2.331153645102946`, β20 → -7.9297637252954285`, β21 → 0.7509718259966994`,
    β22 → 4.621215495379647`, β23 → -3.128768385207422`, β24 → -2.9496406509301445`,
    β25 → 11.654139349330213`, β26 → -2.485237399226773`,
    β27 → -1.6094809637247758`, β28 → 2.4696793745455636`,
    β29 → -5.315151606168697`, β30 → -4.868542637282313`, β31 → -5.001414922460094`,
    β32 → 3.7671472656779583`, β33 → -0.9660086945261992`,
```

$\beta_{34} \to -3.167131194485541`, \beta_{35} \to 5.1943907084546765`,$
$\beta_{36} \to -1.5244941148385052`, \beta_{37} \to 2.0176466222222587`, \beta_{38} \to 5.741432966130173`,$
$\beta_{39} \to -8.743558818673616`, \beta_{40} \to 1.507480589211616`, \beta_{41} \to -3.531369725979072`,$
$\beta_{42} \to 4.347338387744597`, \beta_{43} \to -4.676967851638`, \beta_{44} \to 2.065425059432806`,$
$\beta_{45} \to 0.49007628748992704`, \beta_{46} \to 0.7872755702999652`, \beta_{47} \to 2.265133589779068`,$
$\beta_{48} \to -3.415239488687181`, \beta_{49} \to -14.002389242174646`,$
$\beta_{50} \to 11.46457049328419`, \beta_{51} \to 2.8820654624736624`, \beta_{52} \to 5.150818292716656`,$
$\beta_{53} \to 8.793485100336222`, \beta_{54} \to 4.1959383946917015`, \beta_{55} \to 8.770425918054721`,$
$\beta_{56} \to 3.070291357326339`, \beta_{57} \to -3.1838434788882464`, \beta_{58} \to -2.527916883378694`,$
$\beta_{59} \to -3.185194556547384`, \beta_{60} \to -1.4035148367088757`,$
$\beta_{61} \to -5.7042631521387275`, \beta_{62} \to -2.295160699956549`,$
$\beta_{63} \to -2.557686979572674`, \beta_{64} \to -3.21616753505552`, \beta_{65} \to -4.550887609596588`,$
$\beta_{66} \to 2.9137279007541936`, \beta_{67} \to -5.120709327135425`, \beta_{68} \to 3.74977461411477`,$
$\beta_{69} \to -3.44648620097`, \beta_{70} \to -2.084017674511224`, \beta_{71} \to 8.791272555047744`,$
$\beta_{72} \to 3.996940411578328`, \beta_{73} \to 4.890359222393288`, \beta_{74} \to 2.794181472777685`,$
$\beta_{75} \to 3.8608340220086053`, \beta_{76} \to -7.2152717079492215`, \beta_{77} \to 6.197236238852177`,$
$\beta_{78} \to -3.7534636710769242`, \beta_{79} \to -1.8023701259680356`,$
$\beta_{80} \to -3.2560351613430956`, \beta_{81} \to -8.079341335184084`,$
$\beta_{82} \to -2.7275308244761147`, \beta_{83} \to 7.255186616831219`, \beta_{84} \to -4.205025551006777`,$
$\beta_{85} \to -4.621467983288973`, \beta_{86} \to 2.818624936511229`, \beta_{87} \to 8.031187999797158`,$
$\beta_{88} \to 3.851455244089104`, \beta_{89} \to -1.9711080506701988`, \beta_{90} \to 3.6667783601512722`,$
$\beta_{91} \to 4.595649803446952`, \beta_{92} \to -3.8075182721403453`, \beta_{93} \to -8.769957580231598`,$
$\beta_{94} \to 3.7788588962236056`, \beta_{95} \to 1.4197725672478698`, \beta_{96} \to 4.413067790706901`,$
$\beta_{97} \to -3.9414694931966845`, \beta_{98} \to 1.6830351010133098`,$
$\beta_{99} \to -0.3289197613663763`, \beta_{100} \to -2.89526771558426`,$
$\beta_{101} \to -4.821809971344288`, \beta_{102} \to -1.0747189768819105`,$
$\beta_{103} \to -1.6346426720123481`, \beta_{104} \to 7.491442291424484`,$
$\beta_{105} \to -1.1610648486689248`, \beta_{106} \to 0.6853733699714291`,$
$\beta_{107} \to 0.20856110499461109`, \beta_{108} \to -1.5544907766646432`,$
$\beta_{109} \to 3.603052918693093`, \beta_{110} \to -4.4415630790104235`,$
$\beta_{111} \to -3.710646780935092`, \beta_{112} \to -0.40753118185009485`,$
$\beta_{113} \to -1.8454407267701196`, \beta_{114} \to -2.338850976079607`,$
$\beta_{115} \to 1.8721409768505133`, \beta_{116} \to -2.7382006597803166`,$
$\beta_{117} \to -5.195230247874808`, \beta_{118} \to -3.7482274212195437`,$
$\beta_{119} \to 8.766966844438953`, \beta_{120} \to -1.319956813309461`,$
$\beta_{121} \to 3.8145123624984785`, \beta_{122} \to -5.360436980773363`,$
$\beta_{123} \to -7.74720371322256`, \beta_{124} \to -2.7060013050702203`,$
$\beta_{125} \to -1.9175215711369797`, \beta_{126} \to 4.488338995138896`\};$

Now, create two 6-qubit pure state registers, and apply the circuits using them.
(The CreateQureg commands only need to be run once per session, to create the objects)

In[140]:=
```
ψ = CreateQureg @ 6;
ϕ = CreateQureg @ 6;
```

In[142]:=

```
InitZeroState @ ψ;
ApplyCircuit[circuitA /. paramsA, ψ, ϕ ];
(* apply A to input state ψ=|00..0> putting result in ϕ *)
ApplyCircuit[circuitB /. paramsB, ψ];
(* apply B to input state ψ=|00..0> putting result back in ψ *)

Print[
 "The fidelity of the state 'circuitA.|00..0>' versus 'circuitB.|00..0>' is ",
 CalcFidelity[ ψ, ϕ ], "."]
```

```
The fidelity of the state 'circuitA.|00..0>' versus 'circuitB.|00..0>' is 0.999924.
```