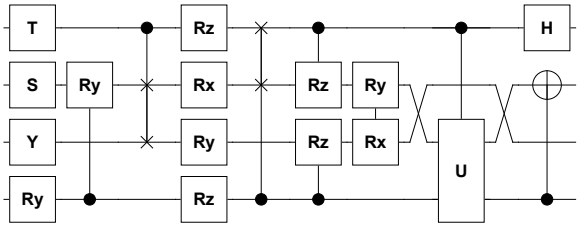
In[1]:=
```
Import["https://quest.qtechtheory.org/QuEST.m"]
CreateDownloadedQuESTEnv[];
```

Create circuits, where **circA |in⟩** will be recompiled to **circB(ϕ) |in⟩**

In[3]:=
```
numQb = 4;
m = 1/√2 {{1, i, 0, 0}, {0, 0, i, 1}, {0, 0, i, -1}, {1, -i, 0, 0}};
circA = Circuit[
    Ry0[.1] Y1 S2 T3 C0[Ry2[.1]] C3[SWAP1,2] ×
    Ry1[.2] Rx2[.3] Rz3[.4] Rz0[.1] C0[SWAP2,3] ×
    C3[Rz2[.1]] C0[Rz1[.2]] R[.1, X1 Y2] ×
    C3[U0,2[m]] C0[X2] H3];

DrawCircuit[circA]
```

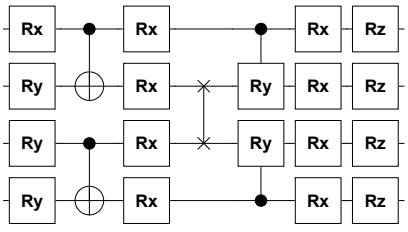
Out[6]=



In[7]:=
```
numϕ = 18 ;
circB = Circuit[
    Ry0[ϕ1] Rx3[ϕ2] Ry1[ϕ3] Ry2[ϕ4] C1[X0] C3[X2] ×
    Rx0[ϕ5] Rx1[ϕ6] Rx2[ϕ7] Rx3[ϕ8] SWAP1,2 C0[Ry1[ϕ9]] C3[Ry2[ϕ10]] ×
    Rx0[ϕ11] Rx1[ϕ12] Rx2[ϕ13] Rx3[ϕ14] ×
    Rz0[ϕ15] Rz1[ϕ16] Rz2[ϕ17] Rz3[ϕ18]];
invB = Reverse[circB];

DrawCircuit[circB, numQb]
```

Out[10]=



Prepare quantum registers and create fictitious Hamiltonian with ground-state **|in⟩ = |+⟩**

In[11]:=
```
{aψ, bψ, hψ, iψ} = CreateQuregs[numQb, 4];
dψ = CreateQuregs[numQb, numϕ];
```

In[13]:=
```
h = -1. X0 - 1. X1 - 1. X2 - 1. X3;
InitPlusState[iψ];
gs = CalcExpecPauliSum[iψ, h, hψ]
```

Out[15]= -4.

Simulate variational recompilation of **circA |in⟩** into **circB(ϕ) |in⟩**

In[16]:=
```
evolve[ϕi_, Δt_, steps_] := Module[
    {ϕvec,ϕrule, matrA,vecC, ϕEvo={},energyEvo={},fidEvo={}},

    InitPureState[aψ, iψ];
    ApplyCircuit[circA, aψ];
    ϕvec=ϕi;
    Do[
        ϕrule=Table[ϕt→ϕvec[[t]],{t,numϕ}];
        CalcQuregDerivs[invB,aψ,ϕrule,dψ];
        matrA=Re @ CalcInnerProducts[dψ];

        ApplyCircuit[invB /. ϕrule, InitPureState[bψ, aψ]];
        ApplyPauliSum[bψ,h,hψ];
        vecC=-Re @ CalcInnerProducts[hψ, dψ];

        ϕvec = ϕvec + Δt LinearSolve[matrA,vecC, Method->"Krylov"];

        AppendTo[ϕEvo,ϕvec];
        AppendTo[energyEvo, CalcExpecPauliSum[bψ,h,hψ]];
        AppendTo[fidEvo, CalcFidelity[bψ,iψ]];
        Null,steps
    ];

    {ϕEvo,energyEvo,fidEvo}
]
```

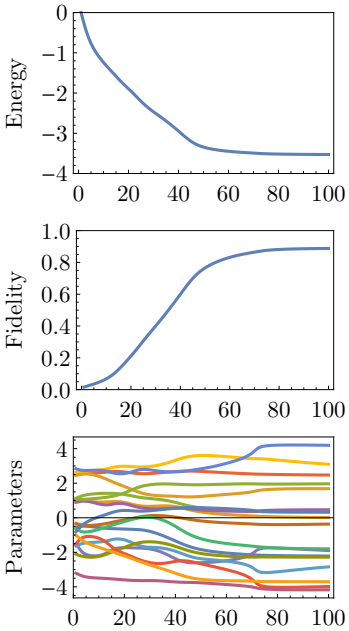
In[17]:=
```
style = {
    Frame→True,FrameStyle→Black,FrameTicks→{{Automatic,None},{Automatic,None}},
    LabelStyle→{FontFamily→"CMU Serif",FontSize→12},
    PlotRange→ {{0,All},Automatic}, ImageSize→Small};
```

... and plot

In[18]:=
```
{ϕEvo, eEvo, fEvo} = evolve[RandomReal[{-π, π}, numϕ], .05, 100];

Column[{
  ListLinePlot[eEvo, PlotRange → {0, gs}, FrameLabel → "Energy", style],
  ListLinePlot[fEvo, PlotRange → {0, 1}, FrameLabel → "Fidelity", style],
  ListLinePlot[Transpose[ϕEvo], FrameLabel → "Parameters", style]
 }]
```

Out[19]=



These discovered parameters produce $\mathbf{B(\phi)|0\rangle \approx A|0\rangle}$

In[20]:=
```
InitPureState[aψ, iψ];
ApplyCircuit[circA, aψ];

InitPureState[bψ, iψ];
Table[ϕt → -ϕEvo[[-1, t]], {t, numϕ}]
ApplyCircuit[circB /. %, bψ];

CalcFidelity[aψ, bψ]
```

Out[23]=
$$\{\phi_1 \to 2.21505,\ \phi_2 \to -0.0145617,\ \phi_3 \to -0.327275,\ \phi_4 \to -2.47312,\ \phi_5 \to 1.89714,\ \phi_6 \to 0.364757,\ \phi_7 \to 2.84149,\ \phi_8 \to -3.09891,\ \phi_9 \to -0.463796,\ \phi_{10} \to 2.27968,\ \phi_{11} \to 3.98071,\ \phi_{12} \to -4.19475,\ \phi_{13} \to 3.70348,\ \phi_{14} \to 4.17053,\ \phi_{15} \to 1.79892,\ \phi_{16} \to -0.336933,\ \phi_{17} \to -1.68437,\ \phi_{18} \to -1.96589\}$$

Out[25]= 0.887947